the failure envelope surface of the standard cap model, the scalar parameter $a$ must be replaced
$\alpha - N$ in the definition $F_e$.

The cap model contains a number of parameters which must be chosen to represent a
particular material, and are generally based on experimental data. The parameters $\alpha$, $\beta$, $\theta$, and $\gamma$ are usually evaluated by fitting a curve through failure data taken from a set of triaxial
compression tests. The parameters $W$, $D$, and $X_0$ define the cap hardening law. The value $W$
represents the void fraction of the uncompressed sample and $D$ governs the slope of the initial
loading curve in hydrostatic compression. The value of $R$ is the ration of major to minor axes of
the quarter ellipse defining the cap surface. Additional details and guidelines for fitting the cap
model to experimental data are found in [Chen and Baladi, 1985].

**Material Model 26: Crushable Foam**

This orthotropic material model does the stress update in the local material system
denoted by the subscripts, $a$, $b$, and $c$. The material model requires the following input
parameters:

- $E$, Young’s modulus for the fully compacted material;
- $\nu$, Poisson’s ratio for the compacted material;
- $\sigma_y$, yield stress for fully compacted honeycomb;
- LCA, load curve number for sigma-aa versus either relative volume or volumetric
  strain (see Figure 19.26.1.);
- LCB, load curve number for sigma-bb versus either relative volume or volumetric
  strain (default: LCB = LCA);
- LCC, the load curve number for sigma-cc versus either relative volume or
  volumetric strain (default: LCC = LCA);
- LCS, the load curve number for shear stress versus either relative volume or
  volumetric strain (default LCS = LCA);
- $V_f$, relative volume at which the honeycomb is fully compacted;
- $E_{au}$, elastic modulus in the uncompressed configuration;
- $E_{bu}$, elastic modulus in the uncompressed configuration;
- $E_{cu}$, elastic modulus in the uncompressed configuration;
- $G_{au}$, elastic shear modulus in the uncompressed configuration;
- $G_{bu}$, elastic shear modulus in the uncompressed configuration;
- $G_{cu}$, elastic shear modulus in the uncompressed configuration;
- LCAB, load curve number for sigma-ab versus either relative volume or volumetric
  strain (default: LCAB = LCS);
- LCBC, load curve number for sigma-bc versus either relative volume or volumetric
  strain default: LCBC = LCS);
- LCCA, load curve number for sigma-ca versus either relative volume or volumetric
  strain (default: LCCA = LCS);
- LCSR, optional load curve number for strain rate effects.
The behavior before compaction is orthotropic where the components of the stress tensor are uncoupled, i.e., an a component of strain will generate resistance in the local a direction with no coupling to the local b and c directions. The elastic moduli vary linearly with the relative volume from their initial values to the fully compacted values:

\[
\begin{align*}
E_{aa} &= E_{aa} + \beta(E - E_{aa}) \\
E_{bb} &= E_{bb} + \beta(E - E_{bb}) \\
E_{cc} &= E_{cc} + \beta(E - E_{cc}) \\
G_{ab} &= G_{ab} + \beta(G - G_{ab}) \\
G_{bc} &= G_{bc} + \beta(G - G_{bc}) \\
G_{ca} &= G_{ca} + \beta(G - G_{ca})
\end{align*}
\]

where

\[
\beta = \max \left[ \min \left( \frac{1-V_{\min}}{1-V}, 1 \right), 0 \right]
\]

and \(G\) is the elastic shear modulus for the fully compacted honeycomb material

\[
G = \frac{E}{2(1+\nu)}
\]

The relative volume \(V\) is defined as the ratio of the current volume over the initial volume; typically, \(V = 1\) at the beginning of a calculation. The relative volume, \(V_{\min}\), is the minimum value reached during the calculation.

The load curves define the magnitude of the average stress as the material changes density (relative volume). Each curve related to this model must have the same number of points and the same abscissa values. There are two ways to define these curves: as a function of relative volume \(V\), or as a function of volumetric strain defined as:

\[
\varepsilon_v = 1 - V
\]

In the former, the first value in the curve should correspond to a value of relative volume slightly less than the fully compacted value. In the latter, the first value in the curve should be less than or equal to zero corresponding to tension and should increase to full compaction. When defining the curves, care should be taken that the extrapolated values do not lead to negative yield stresses.
At the beginning of the stress update we transform each element’s stresses and strain rates into the local element coordinate system. For the uncompact ed material, the trial stress components are updated using the elastic interpolated moduli according to:

\[
\begin{align*}
\sigma_{aa}^{n+1\text{trial}} &= \sigma_{aa}^n + E_{aa} \Delta \epsilon_{aa} \\
\sigma_{bb}^{n+1\text{trial}} &= \sigma_{bb}^n + E_{bb} \Delta \epsilon_{bb} \\
\sigma_{cc}^{n+1\text{trial}} &= \sigma_{cc}^n + E_{cc} \Delta \epsilon_{cc} \\
\sigma_{ab}^{n+1\text{trial}} &= \sigma_{ab}^n + 2G_{ab} \Delta \epsilon_{ab} \\
\sigma_{bc}^{n+1\text{trial}} &= \sigma_{bc}^n + 2G_{bc} \Delta \epsilon_{bc} \\
\sigma_{ca}^{n+1\text{trial}} &= \sigma_{ca}^n + 2G_{ca} \Delta \epsilon_{ca} = 1
\end{align*}
\] (19.26.5)

Then we independently check each component of the updated stresses to ensure that they do not exceed the permissible values determined from the load curves, e.g., if

\[
|\sigma_{ij}^{n+1\text{trial}}| > \lambda \sigma_{ij}^n (V_{\text{min}})
\]

then

\[
\sigma_{ij}^{n+1} = \sigma_{ij}^n (V_{\text{min}}) \frac{\lambda \sigma_{ij}^{n+1\text{trial}}}{|\sigma_{ij}^{n+1\text{trial}}|}
\] (19.26.6)

The parameter \(\lambda\) is either unity or a value taken from the load curve number, LCSR, that defines \(\lambda\) as a function of strain rate. Strain rate is defined here as the Euclidean norm of the deviatoric strain rate tensor.

For fully compacted material we assume that the material behavior is elastic-perfectly plastic and updated the stress components according to

\[
\sigma_{ij}^{n+1\text{trial}} = \sigma_{ij}^n + 2G \Delta \epsilon_{ij}^{\text{dev}}
\] (19.26.7)

where the deviatoric strain increment is defined as

\[
\Delta \epsilon_{ij}^{\text{dev}} = \Delta \epsilon_{ij} - \frac{1}{3} \Delta \epsilon_{kk} \delta_{ij}
\] (19.26.8)

We next check to see if the yield stress for the fully compacted material is exceeded by comparing

\[
\sigma_{ij}^{n+1\text{trial}} = \left(\frac{1}{2} \sigma_{ij}^{n+1\text{trial}} \sigma_{ij}^{n+1\text{trial}}\right)^{\lambda/2}
\] (19.26.9)
the effective trial stress, to the yield stress \( \sigma_y \). If the effective trial stress exceeds the yield stress, we simply scale back the stress components to the yield surface:

\[
S_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}} S_{ij}^{\text{trial}}
\]  

(19.26.10)

We can now update the pressure using the elastic bulk modulus, \( K \):

\[
p^{n+1} = p^n - K \Delta e_{kk}^{n+1/2}
\]

\[
K = \frac{E}{3(1-2\nu)}
\]  

(19.26.11)

and obtain the final value for the Cauchy stress

\[
\sigma_y^{n+1} = S_{ij}^{n+1} - p^{n+1} \delta_{ij}
\]  

(19.26.12)

After completing the stress update, we transform the stresses back to the global configuration.

**Figure 19.26.1.** Stress quantity versus volumetric strain. Note that the “yield stress” at a volumetric strain of zero is nonzero. In the load curve definition, the “time” value is the volumetric strain and the “function” value is the yield stress.