and

\[
\mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

\[
C_3 = (K - 2G/3)\mathbf{e} \quad C_{33} = (K + 4G/3)
\]

\(G\) and \(K\) stands for the shear and bulk modulus, respectively. This algorithm requires computation of the effective stress hessian. The derivation of this is quite straightforward but the expression for it is rather long and is hence omitted in this report.

**Material Model 134: Viscoelastic Fabric**

The viscoelastic fabric model is a variation on the general viscoelastic Material Model 76. This model is valid for 3 and 4 node membrane elements only and is strongly recommended for modeling isotropic viscoelastic fabrics where wrinkling may be a problem. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. If bending stresses are important, use a shell formulation with Model 76.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

\[
\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau
\]

(19.134.1)

where \(g_{ijkl}(t - \tau)\) is the relaxation function.

If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

\[
g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t}
\]

(19.134.2)

We characterize this in the input by shear modulii, \(G_i\), and decay constants, \(\beta_i\). An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk modulii:

\[
k(t) = \sum_{m=1}^{N} K_m e^{-\beta_m t}
\]

(19.134.3)

**Material Model 139: Modified Force Limited**

This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The plastic moment versus rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local \(s\) and \(t\) axes.
Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load versus collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both quantities should be entered as positive for all points, and will be interpreted as compressive. The first point should be (zero, initial collapse load).

The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

Stiffness-proportional damping may be added using the damping factor $\lambda$. This is defined as follows:

$$\lambda = \frac{2 \xi}{\omega}$$

where $\xi$ is the damping factor at the reference frequency $\omega$ (in radians per second). For example if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2 \pi \times 2} = 0.001592$$

If damping is used, a small timestep may be required. LS-DYNA does not check this so to avoid instability it may be necessary to control the timestep via a load curve. As a guide, the timestep required for any given element is multiplied by $0.3L/c\lambda$ when damping is present ($L$ = element length, $c$ = sound speed).

**Moment Interaction**

Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied.

$$\left(\frac{M_r}{M_{yield}}\right)^2 + \left(\frac{M_s}{M_{yield}}\right)^2 + \left(\frac{M_t}{M_{yield}}\right)^2 \geq 1$$

where,

$$M_r, M_s, M_t$$

= current moments

$$M_{yield}, M_{yield}, M_{yield}$$

= yield moments

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example, $M_{yield}$ in the above formula is given by the input yield moment about the local axis times the input scale factor for the local s-axis. For strain-softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by
and similarly for $M_s$ and $M_t$.

Thereafter the plastic moments will be given by:

$$M_{rp} = \min(M_{curve}, M_{yield})$$

where

- $M_{rp}$ = current plastic moment
- $M_{curve}$ = moment taken from load curve at the current rotation scaled according to the scale factor.

The effect of this is to provide an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus if a member is bent about its local s-axis it will then be weaker in torsion and about its local t-axis. For moment-softening curves, the effect is to trim off the initial peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with the current axial load, but it is possible to make hinge formation a function of axial load and subsequent plastic moment a function of the moment at the time the hinge formed. This is discussed in the next section.

**Independent plastic hinge formation**

In addition to the moment interaction equation, Cards 7 through 18 allow plastic hinges to form independently for the s-axis and t-axis at each end of the beam and also for the torsional axis. A plastic hinge is assumed to form if any component of the current moment exceeds the yield moment as defined by the yield moment vs. axial force curves input on cards 7 and 8. If any of the 5 curves is omitted, a hinge will not form for that component. The curves can be defined for both compressive and tensile axial forces. If the axial force falls outside the range of the curve, the first or last point in the curve will be used. A hinge forming for one component of moment does not effect the other components.

Upon forming a hinge, the magnitude of that component of moment will not be permitted to exceed the current plastic moment. The current plastic moment is obtained by interpolating between the plastic moment vs. plastic rotation curves input on cards 10, 12, 14, 16, or 18. Curves may be input for up to 8 hinge moments, where the hinge moment is defined as the yield moment at the time that the hinge formed. Curves must be input in order of increasing hinge moment and each curve should have the same plastic rotation values. The first or last curve will be used if the hinge moment falls outside the range of the curves. If no curves are defined, the plastic moment is obtained from the curves on cards 4 through 6. The plastic moment is scaled by the scale factors on lines 4 to 6.

A hinge will form if either the independent yield moment is exceeded or if the moment interaction equation is satisfied. If both are true, the plastic moment will be set to the minimum of the interpolated value and $M_{rp}$.
Figure 19.139.1. The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.