\[ \Delta e^{ij}_h = q_{ik} q_{jl} \Delta e^{kl}_{loc} \]  

(19.179.24)

which is used to increment the damage tensor \( \varepsilon^{ij}_h \). The factor \( \xi_i \) is now given by

\[ \xi_i = \max(0,1 - \frac{q_{ik} q_{jl} \varepsilon_{h}^{kl}}{0.0001 + \varepsilon_m}) \]  

(19.179.25)

where the quantity \( \varepsilon_m \) in the orthotropic case is the maximum compressive principal strain in any direction during the simulation thus far. As for the isotropic case, the material is completely damaged after one load cycle and reloading will follow load curve \( f_s \). In addition, the directions corresponding to no loading will remain unaffected.

The factors \( \xi \) and \( \xi_i \) are treated as constants in the determination of the tangent stiffness so the contribution is regarded as hyperelastic and follows the exposition given in Section 19.179.1.

The reason for not differentiating the coefficients \( \kappa \), \( \xi \) and \( \xi_i \) is that they are always non-differentiable. Their changes depend on whether the material is loaded or unloaded, i.e., the direction of the load. Even if they were differentiable their contributions would occasionally result in a non-symmetric tangent stiffness matrix and any attempt to symmetrize this would probably destroy its properties. After all, we believe that the one-dimensional nature and simplicity of this foam will be enough for good convergence properties even without differentiating these coefficients.

**Material Model 181: Simplified Rubber Foam**

Material type 181 in LS-DYNA is a simplified “quasi”-hyperelastic rubber model defined by a single uniaxial load curve or by a family of curves at discrete strain rates. The term “quasi” is used because there is really no strain energy function for determining the stresses used in this model. However, for deriving the tangent stiffness matrix we use the formulas as if a strain energy function were present. In addition, a frequency independent damping stress is added to model the energy dissipation commonly observed in rubbers.

**19.181.1 Hyperelasticity Using the Principal Stretch Ratios**

A hyperelastic constitutive law is determined by a strain energy function that here is expressed in terms of the principal stretches, i.e., \( W = W(\lambda_1, \lambda_2, \lambda_3) \). To obtain the Cauchy stress \( \sigma_{ij} \), as well as the constitutive tensor of interest, \( C_{ijkl}^{TC} \), they are first calculated in the principal basis after which they are transformed back to the “base frame”, or standard basis. The complete set of formulas is given by Crisfield [1997] and is for the sake of completeness recapitulated here.

The principal Kirchhoff stress components are given by

\[ \tau_{ii}^E = \lambda_i \frac{\partial W}{\partial \lambda_i} \]  

(19.181.1)

that are transformed to the standard basis using the standard formula
\[ \tau_{ij} = q_{ik} q_{ji} \tau_{ii}^E, \]  
\hspace{2cm} (19.181.2)

The \( q_{ij} \) are the components of the orthogonal tensor containing the eigenvectors of the principal basis. The Cauchy stress is then given by

\[ \sigma_{ij} = J^{-1} \tau_{ij}, \]  
\hspace{2cm} (19.181.3)

where \( J = \lambda_1 \lambda_2 \lambda_3 \) is the relative volume change.

The constitutive tensor that relates the rate of deformation to the Truesdell (convected) rate of Kirchhoff stress can in the principal basis be expressed as

\[ C_{ij}^{TK} = \lambda_j \frac{\partial \tau_{ii}^E}{\partial \lambda_j} - 2 \tau_{ii}^E \delta_{ij}, \]  
\hspace{2cm} (19.181.4)

\[ C_{ij}^{TK} = \lambda_i^2 \tau_{ii}^E - \lambda_i^2 \tau_{jj}^E, \quad i \neq j, \lambda_i \neq \lambda_j \quad \text{(no sum)} \]  
\hspace{2cm} (19.181.4)

\[ C_{ij}^{TK} = \frac{\lambda_i}{2} \left( \frac{\partial \tau_{ii}^E}{\partial \lambda_i} - \frac{\partial \tau_{jj}^E}{\partial \lambda_j} \right), \quad i \neq j, \lambda_i = \lambda_j \]

These components are transformed to the standard basis according to

\[ C_{ijkl}^{TK} = q_{ip} q_{jq} q_{kr} q_{ls} C_{pqrs}^{TKE}, \]  
\hspace{2cm} (19.181.5)

and finally the constitutive tensor relating the rate of deformation to the Truesdell rate of Cauchy stress is obtained through

\[ C_{ijkl}^{TC} = J^{-1} C_{ijkl}^{TK}, \]  
\hspace{2cm} (19.181.6)

19.181.2 Stress and Tangent Stiffness

The principal Kirchhoff stress is in material model 181 given by

\[ \tau_{ii}^E = f(\lambda_i) + K(J - 1) - \frac{1}{3} \sum_{k=1}^{n} f(\lambda_k) \]  
\hspace{2cm} (19.181.7)

where \( f \) is a load curve determined from uniaxial data (possibly at different strain rates). Furthermore, \( K \) is the bulk modulus and \( J \) is the relative volume change of the material. This stress cannot be deduced from a strain energy function unless \( f(\lambda) = E \ln \lambda \) for some constitutive parameter \( E \). A consequence of this is that when using the formulas in the previous section the resulting tangent stiffness matrix is not necessarily symmetric. We remedy this by symmetrizing the formulas according to
19.181.3 Two Remarks

The function $f$ introduced in the previous section depends not only on the stretches but for some choices of input also on the strain rate. Strain rate effects complicate things for an implicit analyst and here one also has to take into account whether the material is in tension/compression or in a loading/unloading stage. We believe that it is of little importance to take into account the strain rate effects when deriving the tangent stiffness matrix and therefore this influence has been disregarded.

For the fully integrated brick element we have used the approach in material model 77 to account for the constant pressure when deriving the tangent stiffness matrix. Experiments have shown that this is crucial to obtain a decent implicit performance for nearly incompressible materials.

19.181.3 Modeling of the Frequency Independent Damping

An elastic-plastic stress $\sigma_d$ is added to model the frequency independent damping properties of rubber. This stress is deviatoric and determined by the shear modulus $G$ and the yield stress $\sigma_y$. This part of the stress is updated incrementally as

$$\tilde{\sigma}_d^{n+1} = \sigma_d^n + 2G I^{\text{dev}} \Delta \epsilon$$

(19.181.9)

where $\Delta \epsilon$ is the strain increment. The trial stress is then radially scaled (if necessary) to the yield surface according to

$$\sigma_d^{n+1} = \tilde{\sigma}_d^{n+1} \min(1, \frac{\sigma_y}{\sigma_{\text{eff}}})$$

(19.181.10)

where $\sigma_{\text{eff}}$ is the effective von Mises stress for the trial stress $\tilde{\sigma}_d^{n+1}$.

The elastic tangent stiffness contribution is given by

$$C_d = 2G I^{\text{dev}}$$

(19.181.11)

and if yield has occurred in the last time step the elastic-plastic tangent is used

$$C_d = 2G I^{\text{dev}} - \frac{3G}{\sigma_y} \sigma_d \otimes \sigma_d.$$ (19.181.12)

Here $I^{\text{dev}}$ is the deviatoric $4^{th}$ order identity tensor.