

## New Feature: Defining Hardening Curve in LS-DYNA®

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### INTRODUCTION

This keyword defines material hardening curve based on a few commonly used material hardening laws. Weighted combinations of the hardening laws are also made possible. The load curve ID, which represents stress-strain curve defined here, can be referenced in the load curve ID used by a specific material model. This feature is applicable to all material models with a hardening curve defined by a load curve using \*DEFINE\_CURVE.

### MAIN FEATURES

With this keyword, a user can define five different types of hardening laws (ITYPE 1 through ITYPE 5), and to combine one of the laws (ITYPE 1) with other laws (ITYPE 2 through ITYPE 5) with weighting factors. There are currently five types of hardening laws implemented as follows,

ITYPE=1: Swift power law in the form of:

$$\sigma = K(e_0 + \epsilon_p)^n,$$

where  $\sigma$  is true effective stress,  $e_0$  is the elastic strain at the initial yield point,  $K$  is a strength coefficient,  $\epsilon_p$  is true effective plastic strain,  $n$  is the work hardening coefficient. Input variables defined as follows:

$$P1 = K, \quad P2 = e_0, P3 = n.$$

ITYPE=2: Voce law in the form of:

$$\sigma = \sigma_0 + R_{sat}(1.0 - e^{-zeta\epsilon_p}),$$

where  $\sigma_0$  is the initial yield stress,  $R_{sat}$  is the stress differential between  $\sigma_0$  and the saturated stress,  $zeta$  is a strain coefficient. Input variables defined as follows:

$$P1 = \sigma_0, P2 = R_{sat}, P3 = zeta, P4: \text{hardening curve contributing weighting factor.}$$

ITYPE=3: Voce law in the form of:

$$\sigma = A - B e^{-C\epsilon_p},$$

where  $A, B, C$  are material constants. Input variables defined as follows:

$$P1 = A, P2 = B, P3 = C, P4: \text{hardening curve contributing weighting factor.}$$

ITYPE=4: Hockett-Sherby law in the form of:

$$\sigma = A - B e^{-C\epsilon_p^H},$$

where  $A, B, C$ , and  $H$  are material constants. Input variables defined as follows:

$P1 = A, P2 = B, P3 = C, P4 = H, P5$ : hardening curve contributing weighting factor.

ITYPE=5: Stoughton-Yoon hardening law in the form of:

$$\sigma = A - B e^{-C * \epsilon_p^m} + D * \epsilon_p,$$

where  $A, B, C, m$  and  $D$  are material constants, and,

$0 < m < 1.0$ , and,

$D \geq 0.0$

Input variables defined as follows:

$P1 = A, P2 = B, P3 = C, P4 = M, P5 = D, P6$ : hardening curve contributing weighting factor.

According to Stoughton-Yoon, “with the exception of metals exhibiting Yield Point Elongation (YPE) effects, this function can represent the stress-strain response for BOTH mild and AHSS steel AND aluminum, from the initial yield point, throughout the small strain range, up to the highest strains realized in bulge tests”.

Note that if  $D = 0.0$ , this function reduces to the Hockett-Sherby law (ITYPE 4). Also note that if  $m = 1.0$  and  $D = 0.0$ , this function reduces to one of the Voce law (ITYPE 3).

ITYPE=11: A weighted combination of ITYPE=1 (first card) and any of the ITYPE=2, 3, or 4 (second card). ITYPE 1 becomes ITYPE 11 when P4 (a contributing weighting factor) is defined in the first card. The variable P4 (for ITYPE 2 and 3), P5 (for ITYPE 4) or P6 (for ITYPE 5) also needs to be defined in the second card, see an example below.

## EXAMPLE

The following example shows a hardening curve (LCID 90903) will be created, which is the combination of 50% of Swift power law (ITYPE 1) and 80% of Hockett-Sherby law (ITYPE 4). In Figure 2, from a single element uniaxial tension test (Figure 1) in LS-DYNA, the hardening curves of the power law (ITYPE 1), Hockett-Sherby law (ITYPE 4) and the weighted combination of the two laws are shown. The weighted combination matches that from the hand-calculation.

```
*MAT_037
$      MID      RO      E      PR      SIGY      ETAN      R      HLCID
      1 7.900E-09 2.070E+05      0.30
*DEFINE_CURVE_STRESS
$-----1-----2-----3-----4-----5-----6-----7-----8
$ ITYPE=1: power law. P1=K, P2=n, P3=e0, P4=0.5.
$ ITYPE=4: Hockett-Sherby law. P1=A, P2=B, P3=C, P4=H, P5=0.8.
$      VOCE: sigma=A-B*exp(-C*eps**H)
$      LCID      TYPE      P1      P2      P3      P4      P5
      90903      11      350.0      0.22      0.01      0.5
      90903      4      162.2      72.2      4.34      1.2      0.8
```

Figure 3 shows an example of Stoughton-Yoon hardening law under uniaxial tension from LS-DYNA, compared with result from hand-calculation.

## REVISION INFORMATION:

This feature is available starting from Revision 113640. ITYPE 5 (Stoughton-Yoon) is available starting from Revision 114803.

## REFERENCE:

LS-DYNA User's Manual (draft).

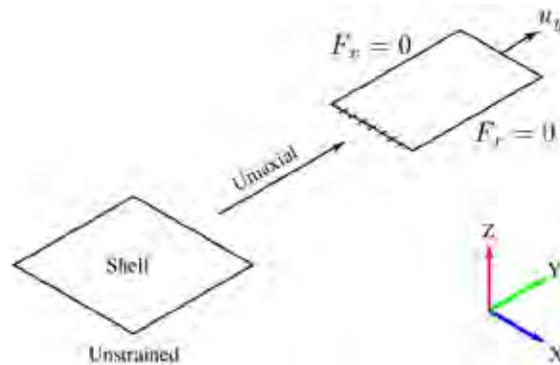


Figure 1 A single element in uniaxial tension

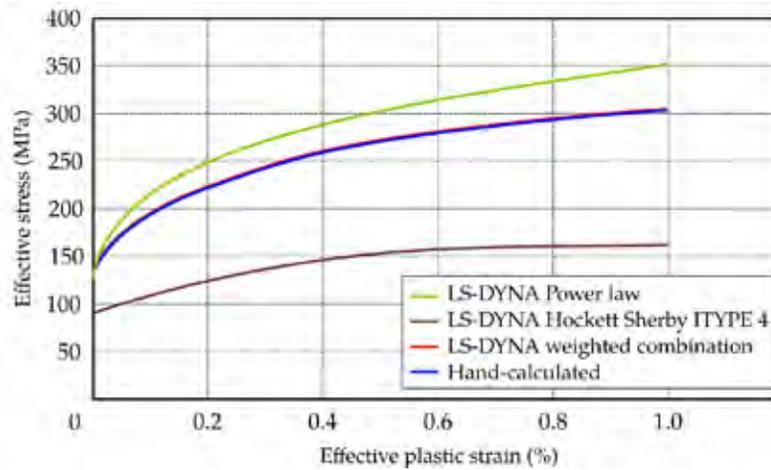
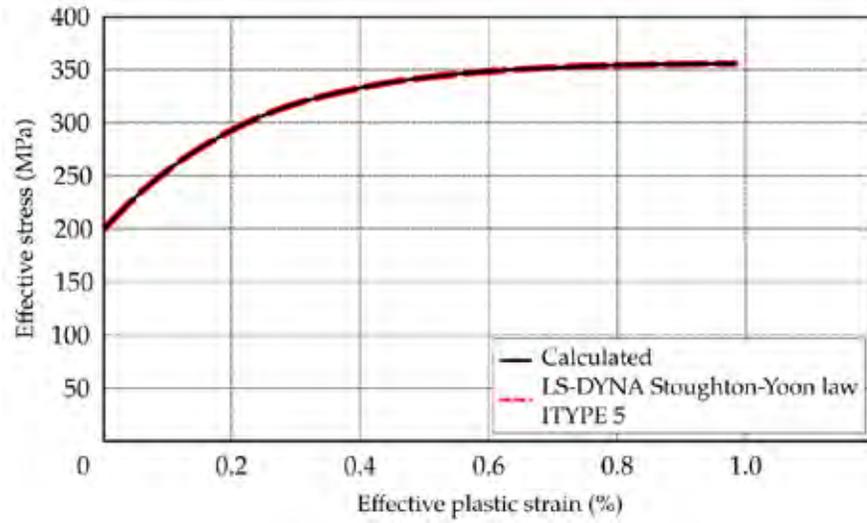


Figure 2 Resulting hardening curves from LS-DYNA comparison with hand-calculation.

$$\text{Power law: } \sigma = 350.0(0.01 + \epsilon_p)^{0.22},$$

$$\text{ITYPE 4 Voce law: } \sigma = 162.2 - 72.2e^{-4.34\epsilon_p^{1.2}},$$

$$\text{Weighted combination: } \sigma = 0.5 * 350.0(0.01 + \epsilon_p)^{0.22} + 0.8(162.2 - 72.2e^{-4.34\epsilon_p^{1.2}}).$$



**Figure 3 Comparison between Stoughton-Yoon results from LS-DYNA and hand- calculation.**

**( $A = 160.8024$ ,  $B = 71.109$ ,  $C = 4.5058$ ,  $M = 0.9989$ ,  $D = 0.8$ .)**